

Axion monodromy inflation with sinusoidal corrections

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Abstract

We study the axion monodromy inflation with a non-perturbatively generated sinusoidal term. The potential form is a mixture between the natural inflation and the axion monodromy inflation potentials. The sinusoidal term is subdominant in the potential, but leaves significant effects on the resultant fluctuation generated during inflation. A larger tensor-to-scalar ratio can be obtained in our model. We study two scenarios, the single inflation scenario and the double inflation scenario. In the first scenario, the axion monodromy inflation with a sufficient number of e-folds generates a larger tensor-to-scalar ratio of about 0.1-0.15 but also a tiny running of the spectral index. In the second scenario of double inflation, axion monodromy inflation is its first stage, and we assume another inflation follows. In this case, our model can realize a larger tensor-to-scalar ratio and a large negative running of the spectral index simultaneously.

I. INTRODUCTION

Recent detection of the gravitational wave perturbation by BICEP2 [1] indicates its large amplitude and its tensor-to-scalar ratio of

$$r_T = 0.20^{+0.07}_{-0.05} \quad (1)$$

for a lensed- Λ CDM plus tensor mode cosmological model, and

$$r_T = 0.16^{+0.06}_{-0.05} \quad (2)$$

after foreground subtraction based on dust models. This implies that the energy scale of inflation is high and the inflation potential would belong to the so-called large field model, where an inflaton takes a super-Planckian field value, such as chaotic inflation [2, 3].

However, it is non-trivial to control a flat potential with a super-Planckian field value. An axion is one of the interesting candidates for inflaton fields, because it has a shift symmetry and its potential would be flat for a super-Planckian field value. In this sense, the so-called natural inflation [4] is interesting, and when studied in light of the BICEP2 data [5]. Furthermore, axion monodromy inflation [6–8] has been proposed as an axion inflation model within the framework of low-energy effective field theory derived from superstring theory (see also [9]).¹ While in natural inflation the potential form is a sinusoidal function, in a simple axion monodromy model the potential is a linear function of the inflaton, $V = a\phi$.²

Axion monodromy inflation with the linear potential is compatible with the data. However, if we examine it in detail, there are a few issues. One is that the linear potential inflation predicts the scalar spectral index $n_s \sim 0.96$ as indicated by PLANCK [14] and $r_T \sim 0.08$. The predicted r_T looks a bit too small compared with BICEP2 results even if we compare with Eq. (2). Another is a tension between BICEP2 and PLANCK, which is a problem not only for this particular model but for most inflation models. BICEP2 reported a somewhat larger tensor-to-scalar ratio than the upper bound $r_T < 0.11$ obtained by PLANCK. As is pointed out in Ref. [1], the tension can be relaxed if the running of the scalar spectral index, α_s , is negatively large as $\alpha_s \sim -(0.02-0.03)$ [14] (see also [15]).

¹ For a review, see [10], and also for recent works [11].

² The potential $V = a\phi^r$ with r being a fractional number is also possible. In addition, such a potential can be also derived by field-theoretical strong dynamics [12] or by non-canonical kinetic terms [13].

In this paper, we study the inflation model driven by the linear potential derived from the axion monodromy inflation model, taking a correction term for non-perturbative effects into account. That is, non-perturbative effects would generate a correction term for the axion in the potential, whose form is the sinusoidal function. Such a correction was mentioned in Ref. [7], but it has been neglected because it is subdominant in the potential. However, we point out non-negligible contributions by the correction term in derivatives of the potential.³ Since the slow roll parameters are expressed in terms of the derivatives of the potential, the resultant correction in the slow roll parameters is quite important in the evaluation of the inflationary observables. Thus, we study the inflation potential with the linear term and sinusoidal term.

Our purpose is two-fold, and we study two scenarios for each of these. First, we point out that the sinusoidal correction term increases the predicted r_T to as much as 0.1-0.15 for the number of e-folds $N = 50$ -60, which corresponds to the cosmological scale $k_* = 0.002 \text{ Mpc}^{-1}$. This size of r_T is within the range of Eq. (2) as reported by BICEP2. We estimate observables n_s , r_T , and α_s for several parameter sets in our model. Next, we study the case where $r_T \simeq 0.1$ -0.2 and the tension between BICEP2 and PLANCK is resolved by a large negative running α_s . The sinusoidal correction term can induce negative running by its potential derivatives; however, the size is not enough as long as we assume that both the horizon and the flatness problem are solved only by this axion monodromy inflation. On the other hand, if we regard this monodromy inflation as the first stage in the double inflation scenario [16] followed by another inflation, we can obtain favored values of the observables $(n_s, r_T, \alpha_s) \sim (0.96, 0.15, -0.02)$. In a previous work, this kind of possibility was studied by two of the present authors (T. K. and O. S.) in the context of the supersymmetric hybrid inflation model [18].⁴

Our paper is organized as follows. In Sect. II, we explain our model explicitly. In Sect. III, we study observables derived from our model in both the single and double inflation scenarios. Section IV is devoted to conclusion and discussion.

³ See also [17], where the same potential was used to explain WMAP seven-year data with $n_s \sim 1.03$ and $\alpha_s \sim -0.03$ without considering tensor-to-scalar ratio.

⁴ A large r_T from double inflation was also studied in Ref. [19].

II. “NATURAL” MONODROMY INFLATION

Before showing our model explicitly, we note the definition of several quantities and show several formulas. We use units with $8\pi G = 1$. The slow-roll parameters for the inflation potential V of the inflaton φ are defined as

$$\eta = \frac{V_{\varphi\varphi}}{V}, \quad (3)$$

$$\varepsilon = \frac{1}{2} \left(\frac{V_{\varphi}}{V} \right)^2, \quad (4)$$

$$\xi = \frac{V_{\varphi} V_{\varphi\varphi\varphi}}{V^2}, \quad (5)$$

where V_{φ} denotes the first derivative of V , and $V_{\varphi\varphi}$ and $V_{\varphi\varphi\varphi}$ have similar meanings. The power spectrum of the density perturbation is given by

$$\mathcal{P}_{\zeta} = \left(\frac{H^2}{2\pi|\dot{\varphi}|} \right)^2 = \frac{V}{24\pi^2\varepsilon}. \quad (6)$$

In the following analysis, a parameter which represents the height of the scalar potential is normalized by this with the amplitude of the temperature anisotropy of the cosmic microwave background radiation. The scalar spectral index, its running and the tensor-to-scalar ratio are expressed as

$$n_s = 1 + 2\eta - 6\varepsilon, \quad (7)$$

$$\alpha_s = 16\varepsilon\eta - 24\varepsilon^2 - 2\xi, \quad (8)$$

$$r_T = 16\varepsilon, \quad (9)$$

respectively by use of the above slow-roll parameters. Also the number of e-folds is evaluated as

$$N = \int_{\varphi_e}^{\varphi} \frac{V}{V_{\varphi}} d\varphi, \quad (10)$$

where φ at the upper bound of the integral is the field value when the number of e-folds is N counting from the end of inflation, and φ_e denotes the field value at the end time of inflation.

For $n_s \sim 0.96$ and $r_T \sim 0.16$, we find $\varepsilon \sim 0.01$ and $\eta \sim 0.01$ from Eqs. (7) and (9). Then, with vanishing ξ , we expect $\alpha_s = \mathcal{O}(0.001)$. Thus, ξ is important to realize a large negative value of α_s , and $\xi = \mathcal{O}(0.01)$ is required for $|\alpha_s| = \mathcal{O}(0.01)$.

Now we write the inflation potential of our model,

$$V = a_1\phi + a_2 \cos\left(\frac{\phi}{f} + \delta\right) + v_0, \quad (11)$$

where a_1 , a_2 , f , δ and v_0 are constants. The first linear term of ϕ would appear in the large ϕ limit from the term $\mu\sqrt{1 + (\phi/M)^2}$, which could originate from the Dirac-Born-Infeld action [6, 7]. We assume that the second term is generated by non-perturbative effects. Indeed, this was also mentioned in Ref. [7], but it has been neglected because it may be subdominant. However, as we will show, even if it is subdominant in V , it would be important in the derivatives, in particular the second and third derivatives of the potential. Thus, we consider both the first and second term. We have added the constant v_0 such that $V = 0$ at the potential minimum. However, when the second term is subdominant in the potential, we can see that the constant v_0 is neglected. Using this potential, we will study the single and double inflation scenarios in the next section.

III. RESULTS

A. Single inflation scenario

Here, we consider the case that the axion monodromy inflation expands enough to solve the horizon and flatness problem and study other observables. Let us call this case the single inflation scenario, where the cosmological scale corresponds to $N = 50$ -60. First of all, the limit with $a_2 \rightarrow 0$ is quite simple. We obtain

$$\varepsilon = \frac{1}{2\phi^2}, \quad \eta = \xi = 0, \quad (12)$$

and

$$N = \frac{1}{2}(\phi^2 - \phi_e^2). \quad (13)$$

Then, when $\phi \sim 10$, we estimate

$$N \sim 50, \quad n_s \sim 0.97, \quad r_T \sim 0.08, \quad (14)$$

which are consistent with PLANCK and BICEP2 (see, e.g., [20]). In addition, we estimate the running as $\alpha_s \sim -0.0006$. A large negative value of α_s is not realized in this case. That

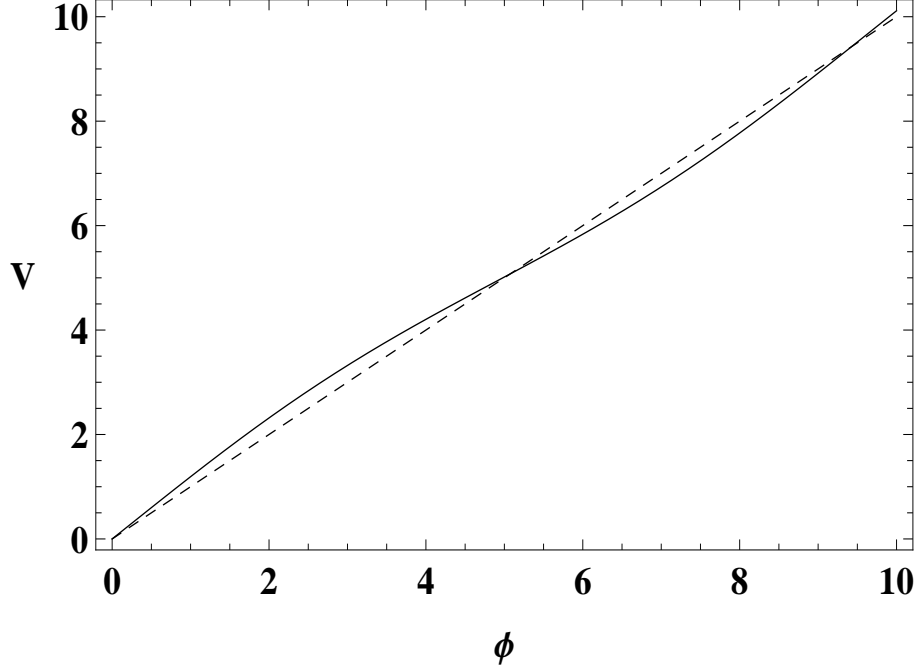


FIG. 1: The shape of the scalar potential where the vertical axis is in an arbitrary unit and rescaled by a_1 . For this figure, we take $A = 0.3$, $\delta = -1.7$, and $x = 0.2$. The solid and dashed lines represent with and without the sinusoidal correction, respectively.

is obvious because in this case we can write

$$n_s - 1 = -\frac{3}{2N}, \quad r_T = \frac{4}{N}, \quad \alpha_s = -\frac{3}{2N^2}. \quad (15)$$

We would have a large negative α_s for smaller N . However, in this case, n_s becomes too small.

Next, we consider the potential with non-vanishing a_2 . Then, the slow-roll parameters can be written as

$$\varepsilon = \frac{1}{2\phi^2} \left(\frac{1 - x \sin \theta}{1 + A\phi^{-1} \cos \theta} \right)^2, \quad (16)$$

$$\eta = -\frac{x^2}{A\phi} \frac{\cos \theta}{(1 + A\phi^{-1} \cos \theta)}, \quad (17)$$

$$\xi = -\frac{x}{A} \sqrt{2\varepsilon} \eta \tan \theta, \quad (18)$$

where $A = a_2/a_1$, $\theta = \phi/f + \delta$, and $x = A/f$. We show the modification of axion monodromy inflation by including the non-perturbative correction of the sinusoidal term in Fig. 1.

For large ϕ , we could approximate

$$\varepsilon \approx \frac{1}{2\phi^2} (1 - x \sin \theta)^2, \quad \eta \approx -\frac{x^2 \cos \theta}{A\phi}. \quad (19)$$

For example, when $-x \sin \theta = 0.4$, we obtain $(1 - x \sin \theta)^2 \approx 2$. Then, we have a factor of 2 enhancement on ε , which results in that on r_T as well, for a fixed value of ϕ . Even for $-x \sin \theta = 0.1$, we would have significant shifts on ε and r_T . On the other hand, as ε becomes larger, n_s becomes smaller than 0.96. We have to cancel such a shift of n_s by increasing η to keep $n_s \simeq 0.96$. Thus, the case with $\cos \theta = 0$ is not realistic, but both $\cos \theta$ and $\sin \theta$ must be of $\mathcal{O}(0.1)$ - $\mathcal{O}(1)$. It is also favorable that both $\cos \theta$ and $\sin \theta$ are negative. In this case, ξ is always negative, thus α_s are mostly positive and can be only tiny negative.⁵ From the above reasoning, it is impossible to realize $N \sim 50$, $n_s \sim 0.96$, $r_T \sim 0.1$ and a large negative α_s simultaneously. We show some examples in Table I and Fig. 2, where we fix $\phi = 10$ and δ can be obtained as $\delta = \theta - \phi/f$.

	x	A	ϕ	θ	n_s	r_T	N	α_s
S1	0.1	0.2	10	$47\pi/36$	0.969	0.098	50	-0.00015
S2	0.2	0.3	10	$19\pi/12$	0.951	0.11	50	0.00046
S3	0.2	0.4	10	$23\pi/18$	0.969	0.12	51	0.00048
S4	0.3	0.4	10	$53\pi/36$	0.957	0.13	55	0.0028
S5	0.3	0.5	10	$53\pi/36$	0.951	0.14	51	0.0013
S6	0.3	0.6	10	$47\pi/36$	0.965	0.14	51	0.0012
S7	0.4	0.6	10	$13\pi/9$	0.953	0.15	58	0.0033
S8	0.4	0.7	10	$25\pi/18$	0.955	0.16	55	0.0025

TABLE I: Results in the single inflation scenario

⁵ On the other hand, we start our discussion to derive a large negative α_s at first. Because of $\xi \simeq \frac{\sqrt{2\varepsilon}x^3}{A^2\phi} \sin \theta$, we are required to obtain $\sin \theta > 0$. In this case, we have $(1 - x \sin \theta)^2 < 1$ in Eq.(19). Thus, we obtain smaller r_T than the linear potential without the sinusoidal term, i.e. $r_T < 0.08$. Here, we concentrate on the linear potential with the sinusoidal correction. If we consider the potential term ϕ^p instead of the linear term, the situation would change. For $p \gtrsim 2$, we already obtain $r_T \gtrsim 0.16$ without the sinusoidal term. Again we need $\sin \theta > 0$ to realize a large negative α_s and it has a negative correction on r_T . However, even including such a negative correction, we can realize $r_T \sim 0.1$ for $p \gtrsim 2$.

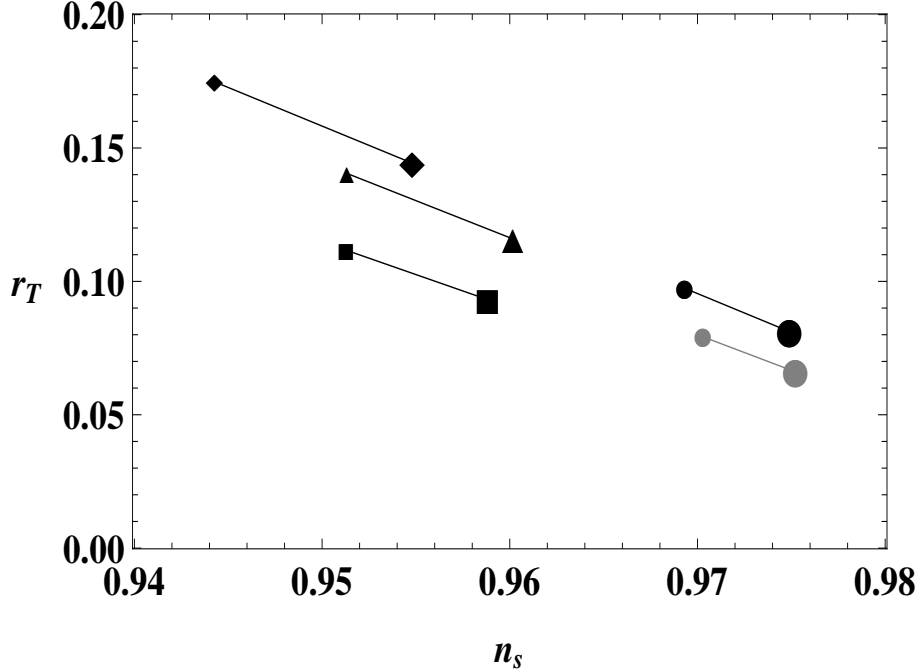


FIG. 2: The predicted n_s and r_T for $N = 50-60$ by some of benchmark parameter sets in Table I. The gray corresponds to the linear potential. The circle, square, triangle, and diamond denote n_s and r_T using x , A , and θ of S1, S2, S4, and S7, respectively. Smaller and larger symbols correspond to $N = 50$ and 60 , respectively.

B. Double inflation scenario

Here, we study the double inflation scenario, where the present cosmological scale corresponds to a smaller number of N in the first monodromy inflation and the second inflation derives another large e-fold of $N \sim 30-50$ to solve the horizon and flatness problems. The second inflation can be any of the small field inflation models, if it generates enough e-folding.

In this scenario, larger ε , and as a result larger r_T , can be obtained because the field value at the k_* mode horizon crossing, ϕ_* is smaller. Since the enhancement of ε can be done with small ϕ , $\sin \theta$ can be positive or negative. To keep $n_s \simeq 0.96$ for larger ε , we need a positive η , which is realized for a negative $\cos \theta$. As mentioned above, the contribution of ξ is important to realize a large negative α_s . For this purpose, ξ must be positive. Then, $\tan \theta$ must be negative. For $\varepsilon \sim 0.01$ and $\eta \sim 0.01$ as well as $A \sim x$, we obtain $2\xi \sim 0.02$ by $\tan \theta \sim \mathcal{O}(10)$. We show some examples in Table II as well as Fig. 3, where N denotes the

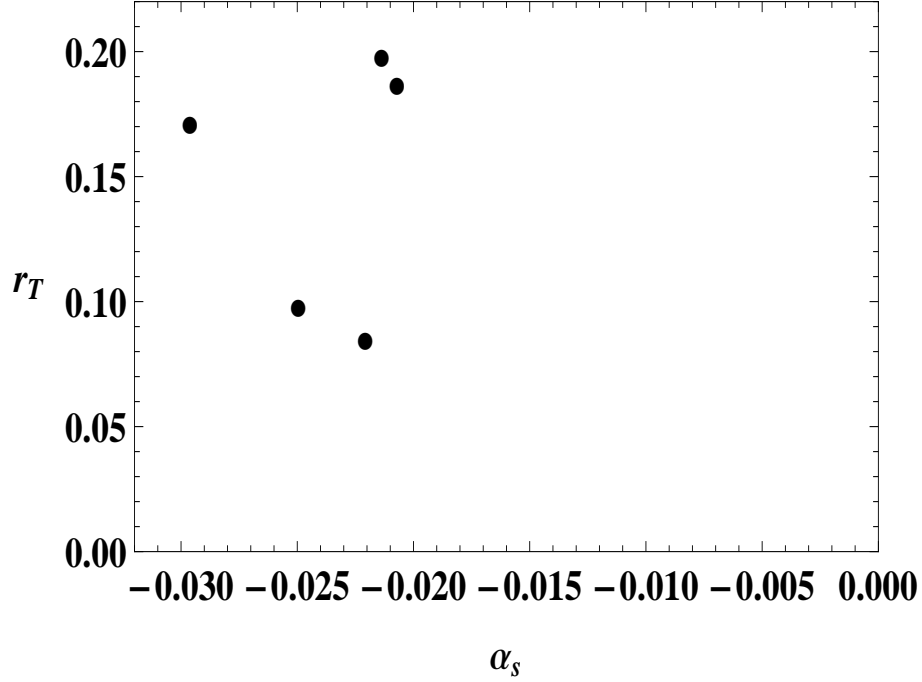


FIG. 3: The predicted α_s and r_T by the benchmark parameter sets in Table II.

e-folding generated through our model at the first stage of inflation.

x	A	ϕ	θ	n_s	r_T	N	α_s
0.2	0.1	7.8	$4\pi/9$	0.951	0.084	31	-0.022
0.3	0.2	6.3	$17\pi/36$	0.951	0.098	21	-0.025
0.3	0.3	4.4	$5\pi/9$	0.953	0.187	12	-0.021
0.4	0.4	3.9	$19\pi/36$	0.953	0.17	11	-0.030
0.4	0.5	4.0	$5\pi/9$	0.955	0.20	11	-0.021

TABLE II: Results in the double inflation scenario

As mentioned above, we assume that the second inflation follows after the above monodromy inflation occurs such that the total number of e-folds becomes 50 or more. In principle, any inflation model can be allowed and there is no constraint at present. Hence, we do not analyze its model-dependent detail here.

IV. SUMMARY

We have studied axion monodromy inflation with a sinusoidal correction term. We find the sinusoidal correction term is important in estimating inflationary observables; in fact, n_s and r_T are significantly affected by the correction. We have shown that a larger tensor-to-scalar ratio compatible with the recent BICEP2 results can be obtained in both the single and double inflation scenarios. A large negative running of the spectral index can be realized in the double inflation scenario, whose first stage is driven by our monodromy inflation model with sinusoidal correction term.

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